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REGRESSION FOR MARKOV BERNOULLI RANDOM VARIABLES.(U)
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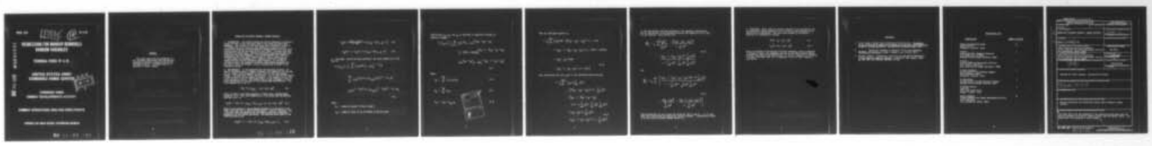

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REGRESSION FOR MARKOV BERNOULLI RANDOM VARIABLES

TECHNICAL PAPER TP 4-78

UNITED STATES ARMY COMBINED ARMS CENTER

COMBINED ARMS COMBAT DEVELOPMENTS ACTIVITY

COMBAT OPERATIONS ANALYSIS DIRECTORATE

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ABSTRACT

This paper deals with the estimation of the regression coefficients when the data are sequences of Bernoulli random variables that form Markov chains. The method used is an extension of Klotz's papers.

REGRESSION FOR MARKOV BERNOULLI RANDOM VARIABLES

1. INTRODUCTION. The problem studied was that of regression on Bernoulli random variables in the case where some of the random variables were dependent. The interest in this case arose from a problem of trying to fit probability of hit curves to data generated by repeated missile simulations performed at US Army Materiel Systems Analysis Activity using tracking data from the Antitank Missile Test (ATMT). Hit/miss data were generated one second apart. Because overlapping tracking data were used, successive shots were dependent. This caused problems that seemed insurmountable until the author became aware of Klotz's papers (1) (2). In these papers the parameters of a sequence of Bernoulli dependent random variables satisfy the Markov chain property. In the case of successive shots, the assumption of Markov chain seemed reasonable and was used to solve the problem. Klotz's technique was extended to the regression problem.

2. PRELIMINARIES. In the generated data the following occurred: for several different ranges, a number of gunners (the number was not the same for all ranges) fired a sequence of shots (not all the same sequence length). The shots were fired a second apart. Let X(I,J,R) be the results of the Ith shot of the Jth gunner at range R. A hit caused X to be 1 and a miss caused it to be 0. The notation that is now introduced is that of Klotz but modified to the needs of the problem under consideration. The first probability of hit is:

$$P(R) = Pr\left\{X_{ijR}\right\} = b_0 + b_1 R + b_2 R^2 \qquad \text{Eq 1}$$

which, as shown in the above equation, is taken to be a second degree polynomial in R. Next, the probability of a hit given that the previous shot was a hit, is:

$$P_{11}(R) = \lambda(R) = Pr\left\{X_{ijR} = 1 | X_{i-1\ jR} = 1\right\} = a_0 + a_1 R + a_2 R^2 \qquad \text{Eq 2}$$

which is also taken as a second degree polynomial in R and which is the lower right hand term in the transition matrix. Clearly, equations 1 and 2 hold only when the sequences are stationary, which was a reasonable assumption for the problem considered. The remaining three terms of the transition matrix are:

$$P_{01}(R) = 1 - \lambda(R) = Pr\left\{X_{ijR} = 0 | X_{i-1\ jR} = 1\right\} \qquad \text{Eq 3}$$

$$P_{10}(R) = \frac{P(R)\left[1 - \lambda(R)\right]}{1 - P(R)} = Pr\left\{ X_{ijR} = 1 \middle| X_{i-1\ jR} = 0 \right\} \qquad \text{Eq 4}$$

$$P_{00}(R) = 1 - P_{10}(R) = Pr\left\{ X_{ijR} = 0 \middle| X_{i-1\ jR} = 0 \right\} \qquad \text{Eq 5}$$

3. **LIKELIHOOD.** Having the above machinery, the joint probability of the data is:

$$Pr\left\{ X_{ijR}\text{'s} \right\} = \underset{R}{\pi} \ \underset{j=1}{\overset{N_R}{\pi}} \left\{ P(R)^{X_{1jR}} \left[1 - P(R)\right]^{1 - X_{1jR}} \underset{i=2}{\overset{n_{jR}}{\pi}} P_{11}(R)^{X_{ijR} X_{i-1\ jR}} P_{10}(R)^{X_{ijR}(1 - X_{i-1\ jR})} P_{01}(R)^{(1 - X_{ijR}) X_{i-1\ jR}} P_{00}(R)^{(1 - X_{ijR})(1 - X_{i-1\ jR})} \right\} \qquad \text{Eq 6}$$

where:

N_R = number of gunners firing at range R

n_{jR} = number of shots by the jth gunner at the Rth range.

Substituting r_{jR}, S_{jR}, and t_{jR} as described in equations 8 through 10, equation 6 becomes:

$$Pr\left\{X_{ijR}'s\right\} = \pi_{R} \; \pi_{j=1}^{N_R} \left\{ \lambda(R)^{r_{jR}} (1 - \lambda(R))^{2(S_{jR} - r_{jR}) - t_{jR}} \left[1 - 2P(R) + \lambda(R)P(R)\right]^{(n_{jR} - 1 - 2S_{jR} + r_{jR} + t_{jR})} P(R)^{(S_{jR} - r_{jR})} \left[1 - P(R)\right]^{-(n_{jR} - 2 - S_{jR} + t_{jR})} \right\}$$

Eq 7

Where:

$$r_{jR} = \sum_{i=2}^{n_{jR}} X_{i-1\;jR} X_{ijR}$$ Eq 8

$$S_{jR} = \sum_{j=1}^{n_{jR}} X_{ijR}$$ Eq 9

$$t_{jR} = X_{1jR} + X_{n_{jR}\;jR}$$ Eq 10

Then the likelihood function is:

$$L = \sum_{R} \sum_{j=1}^{N(R)} \Big\{ r_{jR} \ln\lambda(R) + \left[2(S_{jR} - r_{jR}) - t_{jR}\right] \ln(1 - \lambda(R))$$

$$+ (n_{jR} - 1 - 2S_{jR} + r_{jR} + t_{jR}) \ln(1 - 2P(R) + \lambda(R)P(R))$$

$$+ (S_{jR} - r_{jR}) \ln P(R)$$

$$- (n_{jR} - 2 - S_{jR} + t_{jR}) \ln(1 - P(R)) \Big\} \qquad \text{Eq 11}$$

Now, substituting $\lambda(R)$ and $P_{11}(R)$ in the likelihood function one has:

$$L = \sum_{R} \sum_{j=1}^{N(R)} \Big\{ r_{jR} \ln\left(\sum_{k=0}^{2} a_k R^k\right)$$

$$+ \left[2(S_{jR} - r_{jR}) - t_{jR}\right] \ln\left(1 - \sum_{k=0}^{2} a_k R^k\right)$$

$$+ \left[n_{jR} - 1 - 2S_{jR} + r_{jR} + t_{jR}\right]$$

$$\ln\left(1 - 2\sum_{q=0}^{2} b_q R^q + \sum_{q=0}^{2} b_q R^q \sum_{k=0}^{2} a_k R^k\right)$$

$$+ (S_{jR} - r_{jR}) \ln\left(\sum_{q=0}^{2} b_q R^q\right)$$

$$- (n_{jR} - 2 - S_{jR} + t_{jR}) \ln\left(1 - \sum_{q=0}^{2} b_q R^q\right) \Big\} \qquad \text{Eq 12}$$

To find the maximum likelihood estimates of the regression coefficients, partial derivatives of the likelihood function with respect to the a's and b's are required. These partials are:

$$\frac{\partial L}{\partial a_m} = \sum_R \sum_{j=1}^{N(R)} \left(\frac{r_{jR} R^m}{\sum_{k=0}^{2} a_k R^k} - \frac{\left[2(S_{jR} - r_{jR}) - t_{jR}\right] R^m}{1 - \sum_{k=0}^{2} a_k R^k} \right.$$

Eq 13

$$\left. + \frac{(n_{jR} - 1 - 2S_{jR} + t_{jR})\, R^m \sum_{q=0}^{2} b_q R^q}{1 - 2 \sum_{q=0}^{2} b_q R^q + \sum_{q=0}^{2} b_q R^q \sum_{k=0}^{2} a_k R^k} \right)$$

and

$$\frac{\partial L}{\partial b_n} = \sum_R \sum_{j=1}^{N(R)} \left(\frac{(n_{jR} - 1 - 2S_{jR} + r_{jR} + t_{jR})(-2R^n + R^n \sum_{k=0}^{2} a_k R^k)}{1 - 2 \sum_{q=0}^{2} b_q R^q + \sum_{q=0}^{2} b_q R^q \sum_{k=0}^{2} a_k R^k} \right.$$

Eq 14

$$\left. + \frac{(S_{jR} - r_{jR}) R^n}{\sum_{q=0}^{2} b_q R^q} + \frac{(n_{jR} - 2 - S_{jR} + t_{jR})\, R^n}{1 - \sum_{q=0}^{2} b_q R^q} \right)$$

These expressions are set to zero and solved for the a's and b's. It is clear that the solutions must be obtained by iterative methods. A program was written to do this using the Newton Raphson method (3).

4. CONCLUSION. Recall that the problem discussed in the introduction was the problem of fitting probability of hit curves to data generated by repeated missile simulations. The curves were assumed to be quadratic functions of R expressed as follows:

$$P(R) = b_0 + b_1R + b_2R^2 \qquad \text{Eq 1}$$

$$P_{11}(R) = a_0 + a_1R + a_2R^2 \qquad \text{Eq 2}$$

Hence, utilization of the maximum likelihood technique, given by equations 11 through 14 above, and subsequent solution by the Newton Raphson method, provides the values of the coefficients, a's and b's, necessary to achieving a maximum likelihood "best fit" of equations 1 and 2 to their respective data points.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper deals with the estimation of the regression coefficients when the data are sequences of Bernoulli random variables that form Markov chains. The method used is an extension of Klotz's papers.

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